THE USE OF THE WOODY SCALE FOR DIAGNOSTIC PURPOSES

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So much attention has been given to the measuring value of the various standard scales and tests that another property of such scales of far-reaching importance frequently has been overlooked. It is doubtless true that when these scales are applied to a school system such products as amounts of overlapping of grades, individual variations in ability, grade medians, room medians, etc., should be utilized. Nevertheless, in most instances, the scales have not been used to diagnose the difficulties involved in the processes which they are attempting to measure. From the standpoint of improvement in teaching such a diagnosis can be made of more value to a school system than the usual efficiency rating resulting from the application of these scales. It is precisely for this reason that the writer believes the Freeman Handwriting Scale to be of greater value in the improvement of a pupil's handwriting than either the Ayres or the Thorndike scale. The former indicates which element of the complex process needs attention, the latter measure legibility or quality, both of which are quite complex. If a teacher wishes to secure better results in long division, it is more valuable for her to know what kinds of errors cause a pupil to be weak in this process than it is to know that, measured by standard achievement in division, the pupil belongs in the fifth grade.

It is the purpose of this article to record the results of a study in which the Woody Scale in Division was used, both in rating the division work of pupils and in ascertaining what kind of errors pupils make in division. The second part of this study was made because the teachers were dissatisfied with the results shown by the scale, and wanted to know how the teaching in division could be improved.

The Woody Division Scale, Series A, which follows, was selected because the problems are arranged in the order of their increasing difficulty, and because the steps of difficulty between problems are approximately equal. The tests were given and the papers were scored by two "special-help" teachers, neither of whom has any regular classroom work, and both of whom have had some training in giving and scoring educational tests. The instructions given in every case were as follows:

When ready to give the test, have monitors distribute the test sheets, placing one on each desk, *face downward*. On the blank side of the paper have each pupil write carefully in the upper left-hand corner the date, the name of his school, the name of his town, his teacher's name, and the name of the person

Series A

DIVISION SCALE
By Clifford Woody

Name.....

When is your next birthday?.....How old will you be?.....

Are you a boy or girl?.....In what grade are you?.....

(1) $3\overline{)6}$ (2) $9\overline{)27}$ (3) $34\overline{)28}$ (4) $1\overline{)5}$ (5) $9\overline{)36}$ (6) $3\overline{)39}$

(7) $4 \div 2 =$ (8) $9\overline{)0}$ (9) $1\overline{)1}$ (10) $6 \times \quad = 30$ (11) $2\overline{)13}$ (12) $2 \div 2 =$

(13) $4\overline{)24 \text{ lbs. } 8 \text{ oz.}}$ (14) $8\overline{)5856}$ (15) $\frac{1}{4}$ of $128 =$ (16) $68\overline{)2108}$ (17) $50 \div 7 =$

(18) $13\overline{)65065}$ (19) $248 \div 7 =$ (20) $21.\overline{)252.}$ (21) $25\overline{)9750}$ (22) $2\overline{)13.50}$

(23) $23\overline{)469}$ (24) $75\overline{)2250300}$ (25) $2400\overline{)504000}$ (26) $12\overline{)2.76}$

(27) $\frac{7}{8}$ of $624 =$ (28) $3.\overline{)093.6000}$ (29) $3\frac{1}{2} \div 9 =$ (30) $\frac{3}{4} \div 5 =$

(31) $\frac{5}{4} \div \frac{3}{5} =$ (32) $9\frac{5}{8} \div 3\frac{3}{4} =$ (33) $52\overline{)3756}$

(34) $62.50 \div 1\frac{1}{4} =$ (35) $315\overline{)37722}$ (36) $9\overline{)69 \text{ lbs. } 9 \text{ oz.}}$

giving the test. Follow this order in giving the foregoing information. Next, have the pupils turn the papers over and fill out the blanks at the top of the page, each pupil turning his paper back again promptly as soon as he has finished this. When all are ready for the test, instruct the pupils that, when the signal is given they are to turn the papers over and solve as many problems in twenty minutes as they can, being sure to get them right. Explain that all the problems are division problems. (The problems may be called "into" problems if the pupils are more accustomed to this term.) Instruct the pupils that they are to use this sheet only to figure upon, not to use scratch paper, and to work out the problems just as if they were working them on the blackboard. Pupils should also be told that as the figures on the reverse side are printed they will need to make their figures smaller than usual. If the pupils are not familiar with the division form used by the author, this should be explained and the pupils told they may use any form to which they are accustomed. Be sure to allow *exactly* twenty minutes. Have pupils hold up their pencils when the signal "Stop" is given. If any irregularities occur, as they unavoidably do at times, these should be noted, the paper or set of papers marked, and sent in with the rest

The method of scoring the problems is indicated in Table I. The vertical column at the extreme left contains the serial numbers of the problems. The horizontal column at the top indicates the pupils by numbers. Under the numbers of the pupils are found black crosses indicating that the problem was worked correctly, black ovals indicating an incorrect operation, and blank spaces indicating no attempts. The remaining data are self-explanatory. A similar sheet was used for each room. From these tabulations the data for Tables II and III were obtained.

Table II gives the distribution of pupils' scores for the entire school system. This table is to be read as follows: In the fourth grade one pupil worked 2 problems correctly, one worked 5 problems, three, 6 problems, etc. There were 88 pupils in the fourth grade, 100 in the fifth grade, 93 in the sixth grade, 83 in the seventh grade, and 41 in the eighth grade. The city medians are shown on the bottom line. This table was used as the basis for Figs. 1 and 2.

Figure 1 shows graphically the median for each grade of each school and the medians for the city grades as a whole. The medians of the grades for the entire city show a steady increase up to the seventh grade. There is practically no increase in ability to perform division operations beyond the seventh grade. The South

TABLE I

Subject *Arithmetic* Author *Woody* Series *A* Test *Division* City *Stoughton* Number of papers, *35*
School *Central* Grade *A* Date *Nov. 1, '17* Teacher *Kidney* Scored by *Rynning* Class median, *13*

Number of Questions	Value	Pupils by Number																																								Not Correct	Correct	Distribution of Pupils According to Number Questions Correctly Answered	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	Total	Total		Total
1.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	o	x	o	x	x	x	x	x	o	x	x	x	x	x	x	x	x	o	o	o						7	28	0	
2.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x	x	x	x	o	o						3	32	1	
3.	1	x	x	x	x	x	x	x	o	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	o	x	x	o	o	x	o	x	x	x						6	29	2	
4.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x						0	35	3	
5.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	o	x	x	o	x	o	x						4	31	4	
6.	1	x	x	x	x	o	o	x	o	x	o	o	x	x	x	x	o	x	x	o	o	o	x	o	x	o	x	o	o	o	o	o	o		o	o						19	15	5	1
7.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x	x	x	o	x	x	x	x	x	x	x	x	x	x	o						3	32	6	1
8.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x	x	x						1	34	7	
9.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x		x						1	33	8	4
10.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	o						2	33	9	2
11.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x	x	x	x	x	x	o	o	x	o	x	o	o	o	o	o	x		x	o						10	24	10	1
12.	1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	o	x	x	o	x	x	o	o	x	o	o	o		o	x						8	26	11	3
13.	1	x	x	x	x	x	x	o	x	x	x	x	x	x		x	x	x	o		o	x	x	x	x	x		x		x		x				o						4	23	12	6
14.	1	x	x	x	x	x	x	x	x	o	o	x	x	x		o	x	o	x	x	o	x	o	o	x	x	o	x	o	o		o				o						12	18	13	2
15.	1	x	x	x	x	o	o		x		x	x		o	x				o					o	x					o						o						7	9	14	8
16.	1		o	o			o				o	o	o				o	o	o		o			o	o		o			o						o						15	0	15	3
17.	1	x	x	x	x	x	x	x	o	o	x	o	o		x	x	o	o	x	x	x		o	x	o		o		x	o		o				o						12	15	16	2
18.	1		o				o					o	o				o	o	o								o			o						o						10	0	17	1
19.	1	x	x		o	x	x	x	x	x		o	o		o		o	o	o	o			o						o	o						o						12	7	18	1
20.	1	x	o				o					o			o		o	o	o								o			o						o						10	1	19	
21.	1						o					o					o	o	o								o															6	0	20	
22.	1	o			o	o	o	o	o	o		o	o		o	o	x	o	o				o				o		o													16	1	21	
23.	1						o					o	o		x			o	o								o															6	1	22	
24.	1														o			o	o																							3	0	23	
25.	1															o			o																							2	0	24	
26.	1	o													o				o								o															4	0	25	
27.	1															o																										1	0	26	
28.	1														o	o																										2	0	27	
29.	1															o																										1	0	28	
30.	1															o																										1	0	29	
31.	1																																									0	0	30	
32.	1																																									0	0	31	
33.	1																										o															1	0	32	
34.	1																																									0	0	33	
35.	1																																									0	0	34	
36.	1															o																										1	0	35	
Total not correct		2	3	1	2	3	8	2	4	3	3	9	6	1	7	9	8	10	13	3	4	2	5	5	5	3	11	4	7	11	4	7	4	1	6	14								Total	35
Total correct		18	17	16	16	15	15	15	14	14	14	14	14	14	14	14	13	13	12	12	12	12	12	12	11	11	11	10	9	9	8	8	8	8	5	6									

School is above the city median except in the fourth grade. The West School follows closely the city median; the Central School falls below in the sixth grade. The fact that the high

TABLE II

DISTRIBUTION OF PUPILS' SCORES

No. of Problem	Grade				
	IV	V	VI	VII	VIII
1	.....	1	.....	.....	.....
2	1	.....	.....	.....	.....
3	.....	.....	.....	.....	.....
4	.....	.....	.....	.....	.....
5	1	.....	.....	.....	.....
6	3	1	.....	.....	.....
7	1	1	.....	.....	.....
8	11	2	.....	.....	.....
9	7	.....	.....	.....	.....
10	5	9	2	.....	.....
11	12	7	3	.....	.....
12	11	6	.....	.....	.....
13	8	9	1	1	.....
14	13	3	2	.....	1
15	5	2	2	.....	1
16	3	7	4	.....	1
17	1	10	4	.....	.....
18	2	2	4	.....	.....
19	.....	12	2	2	.....
20	.....	6	8	2	2
21	.....	9	6	1	.....
22	.....	8	9	6	.....
23	.....	2	12	1	.....
24	.....	1	11	3	1
25	.....	1	9	2	1
26	.....	1	5	8	1
27	.....	.....	3	3	2
28	.....	.....	3	9	3
29	.....	.....	1	5	3
30	.....	.....	.....	5	4
31	.....	.....	.....	9	3
32	.....	.....	1	15	6
33	.....	.....	.....	5	3
34	.....	.....	1	2	5
35	.....	.....	.....	3	2
36	.....	.....	.....	1	1
Total	88	100	93	83	41
Median	12.3	17.2	22.5	29.8	30.0

medians in the fifth and sixth grades of the South School do not influence the city medians materially is due to the relatively small number of pupils in each grade of this school. This fact may also

account for the high median, for the teachers can give more time per pupil to the grades. In the Central School departmental teaching is maintained in the fourth, fifth, and sixth grades. One

TABLE III

SHOWING NUMBER OF TIMES EACH PROBLEM WAS WORKED INCORRECTLY

No. of Problem	Grade					Total City
	IV	V	VI	VII	VIII	
1	13	13	3	1	1	31
2	8	2	.....	.....	.....	10
3	11	6	3	2	.....	22
4	13	9	5	4	1	32
5	12	.....	2	1	.....	15
6	62	37	11	5	.....	115
7	9	21	5	2	3	40
8	16	6	2	1	3	28
9	11	10	9	10	7	47
10	7	4	8	3	2	24
11	27	24	3	1	5	60
12	26	33	26	21	11	117
13	29	16	8	2	.....	55
14	37	24	12	6	3	82
15	28	27	9	5	1	70
16	26	42	25	14	7	114
17	34	55	22	8	7	126
18	23	50	22	17	11	123
19	43	40	24	21	8	136
20	26	43	61	11	11	152
21	26	53	35	18	7	139
22	45	60	28	13	6	152
23	.....	45	28	14	4	91
24	.....	52	28	19	8	107
25	.....	40	31	29	13	113
26	.....	57	37	18	11	123
27	.....	38	46	19	12	115
28	.....	60	58	26	9	153
29	.....	45	52	43	15	155
30	.....	44	59	41	15	159
31	.....	26	54	45	16	141
32	.....	20	37	32	11	100
33	.....	47	49	28	13	137
34	.....	12	23	47	15	97
35	.....	40	54	29	12	135
36	.....	38	57	66	23	184

would conclude without this graph that departmental teaching in arithmetic should result in better work and a higher median. This conclusion does not hold true. Either the departmental teacher is not efficient, or departmental work in arithmetic is not superior to the traditional grade work.

Figure 2 shows the range of distribution and the overlapping of grades. The heavy broken line shows the city median for each grade. This figure merely corroborates the testimony of every statistical study of this kind. Such wide distribution and grade overlapping are common. However, it was a revelation to the

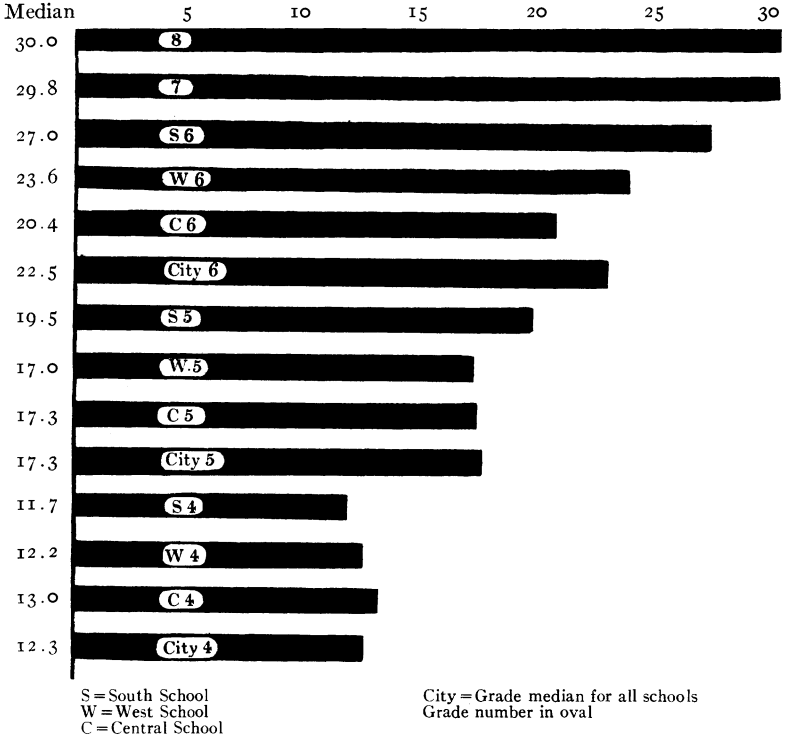

FIG. 1.—Grade medians for each school and for city

teachers of this system, who had not a wide acquaintanceship with statistical investigations. It was the source of many questions and interesting problems for further investigation.

Thus far this study has taken into consideration the data usually regarded as the most significant and valuable. Yet the teachers complained that the questions which were most vital had not been answered. They wished to know why pupils could not work the sixth problem, what difficulty the twelfth problem offered, where

errors in long division occurred, how to narrow the range of distribution, what should be done to correct the overlapping of grades, etc.

In order to answer these questions in part, a study of the types of errors made in working problems was undertaken. Table III shows the number of times each problem was worked incorrectly.

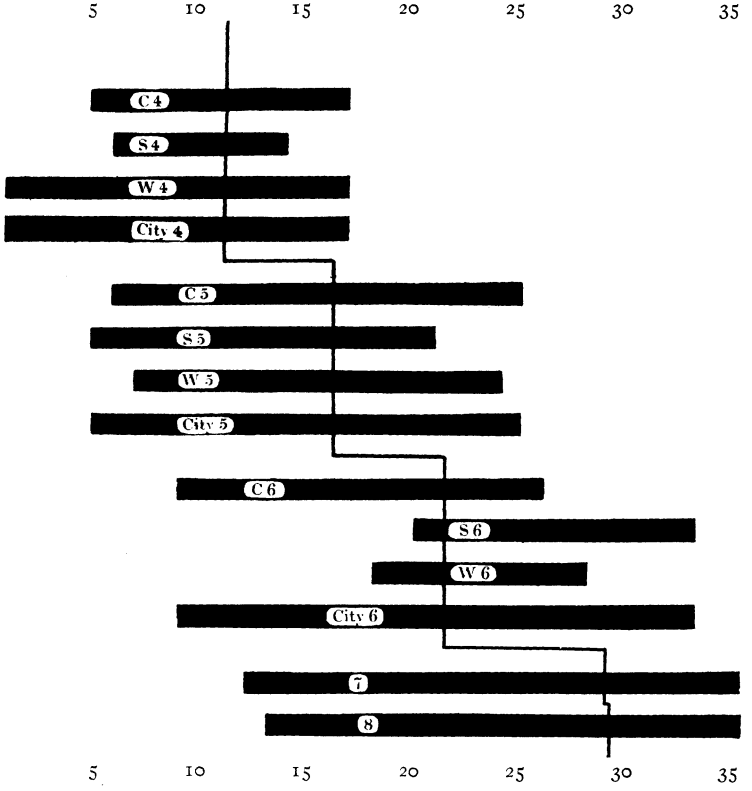

FIG. 2.—Distribution of scores showing overlapping of grades. Broken vertical line shows grade medians.

It is to be read as follows: problem 1 was worked incorrectly by 13 pupils in the fourth grade, 13 pupils in the fifth grade, 3 in the sixth grade, and one each in the seventh and eighth grades, or by 31 pupils in the five upper grades of the system, etc. Such a table is of value in that it shows which problems were the most difficult for the pupils. It is evident at a glance that problems 6, 11, 12, and from 16 on, need investigation.

The sixth problem is $3\overline{)39}$. Some of the answers given are

$$9\tfrac{1}{3}\tfrac{2}{3},\quad 9,\quad 9\tfrac{3}{3},\quad 10,\quad 10\tfrac{3}{3},\quad 10\tfrac{4}{3},\quad 10\tfrac{9}{3},\quad 12,\quad 12\tfrac{2}{3},\quad 12\tfrac{2}{3},\quad 12\tfrac{4}{3}$$

In order to ascertain how these various results were obtained, pupils were given individual oral tests in rooms where no one but pupil and teacher were present. Each pupil was asked to think "aloud," and tell everything he thought and did while solving the problem. It was found that many of the pupils who gave the foregoing answers used the dividend as a unit, ie., found how many times 3 was contained in 39 as a whole instead of saying, first, 3 into 3, then 3 into 9. Those pupils who gave as a result $9\tfrac{1}{3}\tfrac{2}{3}$, 9, $9\tfrac{3}{3}$, did not know the multiplication tables beyond the nines. It was both interesting and instructive to note the mental "twists" which caused errors in individual cases. One pupil gave this result,

$$\begin{array}{r} 1 \text{ with } 6 \text{ remainder} \\ \hline 3\overline{)\ 39} \\ \underline{93} \\ 6 \end{array}$$

His mental procedure was, "3 into $3=1$, $3\times3=9$, and $1\times3=3$." He placed 93 under 39, subtracting the 3 from the 9, getting 6 remainder. Another pupil had the following result:

$$\begin{array}{r} 93 \\ \hline 3\overline{)\ 39} \end{array}$$

He said, "3 into $3=9$, then 3 into $9=3$." This was a case of confusion of the multiplication and the division operation.

Table III shows problem 11 to be a troublesome one also for the fourth and fifth grades. This problem is $2\overline{)13}$. Several pupils gave the following reason for their work:

$$\begin{array}{r} 5-3 \\ \hline 2\overline{)\ 13} \end{array}$$

"Because $5\times2=10$." Their trouble was ignorance of the multiplication table of sixes. Another pupil gave this solution:

$$\begin{array}{r} 11-1 \\ \hline 2\overline{)\ 13} \end{array}$$

"Because 2 into $2=1$, and 2 into $3=1$ with one over." A third pupil gave this solution:

$$2\overline{)\,13}\quad 6\ 2/1$$

He was having difficulty with the remainder. The same type of difficulty caused the following error:

$$2\overline{)\,13}\quad 61$$

From Table III, the twelfth problem is seen to be a source of trouble. The problem appears to be a very simple one, "$2\div 2=\ $." That it is not as easy as one might suppose is evidenced by the fact that the author of the scale placed it twelfth in the order of difficulty. It is not in the form in which pupils are accustomed to see such problems. Investigation with individual pupils reveals the following mental operations: $2\div 2=4$ and $2\div 2=2$. In the first case the error is manifestly one of signs, due to the unaccustomed form. In the second case the pupil said $2\div 2=2$, because $2\times 1=2$.

Table III shows that problem 17 was a stumbling-block. In the majority of cases the problem was not attempted because of the form in which it was presented. Fifteen pupils made errors, because of ignorance of the multiplication tables. One pupil said, "7 and what makes 50," showing that he had confused the division and the subtraction signs. Four pupils placed the dividend under the remainder, instead of using the divisor.

Space will not permit the insertion of any more records of short-division errors. A tabulation of the various types of errors made gives the following distribution:

1. Ignorance of multiplication tables—30 per cent. Illustration: $8\overline{)\,5,856}\quad 8,107$
2. Using dividend as a whole—14 per cent. Illustration: $3\overline{)\,39}\quad 12-3$
3. Confusion of multiplication and division—14 per cent. Illustration:
$$3\overline{)\,39}\quad 93$$
4. Remainder—10 per cent. Illustration: $2\overline{)\,13}\quad 6\ 2/1$
5. Confusion of signs—7 per cent. Illustration: $2\div 2=4$.
6. Form of example strange—5 per cent. Illustration: $\frac{1}{4}$ of 128.

7. Carrying (either forgetting to carry or ignorance of what should be carried)—5 per cent. Illustration: $2\overline{)1{,}350}$ (620)

8. Value of 0—5 per cent. Illustration: $9\overline{)0}$. $1\overline{)1}$ (9 0)

9. Confusion of addition and multiplication—5 per cent. Illustration: $3\overline{)6}$ (3)

10. Confusion of dividend and divisor—2 per cent. Illustration: $8\overline{)498}$ (212)

(This quotient is explained as follows: 4 into 8 = 2, 8 into 9 = 1–1 over, 8 into 18 = 2–2 over.)

11. Using some figure in dividend twice—2 per cent. Illustration: $8\overline{)5{,}856}$ (7,107)

12. Transposing answer—1 per cent. Illustration: $\frac{1}{4}$ of 128 = 23.

Table III shows that problems involving long division were especially fruitful of errors. A separate study of the errors made in working such problems was made. It was found that the errors were as follows:

1. Trial division—38 per cent.
2. Multiplication—28 per cent.
3. Bringing down numbers—16 per cent.
4. Placing wrong figure in quotient or forgetting to place ciphers in quotient—14 per cent.
5. Subtraction—2 per cent.
6. Guess work—2 per cent.

The following example is typical of the first kind of error enumerated above:

$$25\overline{)97{,}050}\quad 4,\ 100$$

In working this problem the pupil said, "25 goes into 97 four times because 2 goes into 9 four times." The multiplication errors need no illustration. The following is an example of errors nos. 3 and 4:

$$75\overline{)2250300}\quad 34,\ 225,\ 300,\ 300$$

This is the twenty-fourth problem in the Woody Division Scale. The pupil in working it said, "75 goes into 225 three times, because 7 goes into 22 three times. 3 times 75 is 225. There is no remainder. Bring down 3. Seventy-five doesn't go into 3 so bring down both ciphers. 75 goes into 300 four times." This illustrates the failure to bring down the first cipher following 5 in the dividend, also to use 3 and 30 as trial dividends. This failure resulted in the omission of two ciphers in the quotient.

Errors in long division are due then to the following causes:

1. The assumption that the first integer of the divisor may be used always as a trial divisor.
2. The trial-and-error method of finding quotient.
3. Ignorance of multiplication tables.
4. Carrying wrong number when multiplying.
5. Borrowing in subtraction.
6. Ignorance of value of cipher.
7. Forgetting to place integers in quotient.

It is, of course, evident that such an analysis of division errors could have been made without the use of the Woody Division Scale. Nevertheless, this study, in conjunction with the results of the test which gave rise to it, has revealed to the teachers of one school system what must be done if the pupils are to carry away with them the ability to use this tool of knowledge accurately and effectively. They have learned how they may become diagnosticians of mathematical diseases, as well as drillmasters.